Stochastic Analysis and Applications Conference Booklet

In Honour of Professor Bijan Z. Zangeneh

November 10-11, 2021

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Schedule, Second Day

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11:00	11:25	**Mazyar Ghani Varzaneh** (Technical University of Berlin & Sharif University of Technology) Multiplicative Ergodic Theorem on the Fields of Banach Spaces
11:30	11:55	**Mir-Omid Haji-Mirsadeghi** (Sharif University of Technology) Doeblin Trees
12:00	18:00	Break
18:00	18:25	**Sima Mehri** (University of Warwick) A Stochastic Gronwall Lemma and Wellposedness of Path-Dependent SDEs Driven by Martingale Noise
18:30	18:55	**Neda Esmaeeli** (University of Isfahan) Pricing of Callable Put Options with and without Extra Information: A Special Case
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نگاهی فلسفی پسا کویدی به ریسک

هیربد آسا

مدرسه بازرگانی کنت

در این سخنرانی کوتاه به طور مختصری به مقوله مدیریت ریسک های بزرگ، فجایع و کلان خواهیم پرداخت. سوال هایی در این راستا مطرح خواهند شد ناظر به مدیریت ریسک تجمیع با وابستگی است. سوال این است که در هنگام شوک های همه گیر، مانند پاندمی، آیا ابزار مدیریت ریسک فرا دست ترجیحی بر ابزار ریسک پیش دست دارد. مهمترین هدف این مقاله بررسی وجود ابزارهای کمی و ریاضی برای بررسی این موضوع است.

متغیرهای تصادفی با وابستگی منفی و قضیه‌ی صفر-یک

کسری علیشاهی

دانشگاه صنعتی شریف

در سال‌های اخیر، گام‌های بلندی در جهت ایجاد یک نظریه‌ی نیرومند برای شناخت و به کارگیری متغیرهای تصادفی با وابستگی منفی برداشته شده است. فرآیندهای دترمینانی، رایلی و لورنتسی، رده‌هایی از فرآیندهای تصادفی با وابستگی منفی هستند که اخیرا معرفی شده‌اند و مورد توجه قرار گرفته‌اند. ساختار غنی جبری/هندسی این فرآیندها امکانات زیادی در اختیار ما قرار داده است و بسیاری از (اما نه همه‌ی) مثال‌های شناخته شده از فرآیندهای با وابستگی منفی را در بر می‌گیرند .در این سخن‌رانی، از چند منظر مختلف به بررسی یک ایده‌ی کلی خواهیم پرداخت: فرآیندهای تصادفی با وابستگی منفی فاصله‌ی محدودی از استقلال دارند! یکی از نمودهای این ایده، تلاش برای تعمیم قضیه‌ی صفر-یک گولموگروف برای چنین متغیرهایی است.

حل عددي معادلات ديفرانسيل تصادفی در ايران

(به مناسب بزرگداشت خدمات علمی و پژوهشی دکتر بیژن ظهوري زنگنه در رشد و توسعه این شاخه در ایران)

علی فروش باستانی

مرکز تحصیلات تکمیلی علوم پایه زنجان

در طول دو دهه گذشته و با توسعه سریع نظریه و کاربردهای معادلات دیفرانسیل تصادفی، نیاز وافری به روش‌های محاسباتی سریع و کارآمد برای مواجهه عملی با این دسته از معادلات در صنعت و مهندسی شکل گرفته است. در این سخنرانی، پس از مروری اجمالی بر کلیت موضوع و مسیر طی شده در 15 سال گذشته (بنا بر تجربه شخصی نگارنده) تلاش می‌شود تصویری از وضعیت تحقیقاتی این شاخه در ایران و تاثیر مکتب علمی دکتر بیژن ظهوری زنگنه در این مسیر ارائه می‌گردد. همچنین اشاره‌ای کوتاه به آینده تحقیقاتی این موضوع و مباحث چالشی مطرح در این زمینه از منظر نگارنده خواهد شد.

اثباتی کوتاه برای قضیه وجود و یک‌تایی جواب SDE

امید نقشینه ارجمند

دانشگاه صنعتی امیرکبیر

این یک سخن‌رانی آموزشی است که در آن به اثباتی برای قضیه وجود و یک‌تایی جواب SDE می‌پردازم که در مقایسه با اثبات ارائه شده در کتاب اکساندل، کوتاه‌تر و سر راست‌تر است.

Probabilistic Mixture Modeling of Scorecards

Hamidreza Arian

Sharif University of Technology

Credit scoring is a rapidly expanding analytical technique used by banks and other financial institutions. Academic studies on credit scoring provide a range of classification techniques used to differentiate between good and bad borrowers. The main contribution of this paper is to introduce a new method for credit scoring based on Gaussian Mixture Models. Our algorithm classifies consumers into groups which are labeled as positive or negative. Labels are estimated according to the probability associated with each class. We apply our model with real world databases from Australia, Japan, and Germany. Numerical results show that not only our model's performance is comparable to others, but also its flexibility avoids over-fitting even in the absence of standard cross validation techniques. The framework developed by this paper can provide a computationally efficient and powerful tool for assessment of consumer default risk in related financial institutions.

A Simple Measure of Conditional Dependence

Mona Azadkia

ETH Zürich

We propose a coefficient of conditional dependence between two random variables Y and Z given a set of other variables $X_1, \dots, X_p$, based on an i.i.d. sample. The coefficient has a long list of desirable properties, the most important of which is that under absolutely no distributional assumptions, it converges to a limit in [0,1], where the limit is 0 if and only if Y and Z are conditionally independent given $X_1, \dots, X_p$ and is 1 if and only if Y is equal to a measurable function of Z given $X_1, \dots, X_p$. Moreover, it has a natural interpretation as a nonlinear generalization of the familiar partial R^2 statistic for measuring conditional dependence by regression. Using this statistic, we devise a new variable selection algorithm, called Feature Ordering by Conditional Independence (FOCI), which is model-free, has no tuning parameters, and is provably consistent under sparsity assumptions. A number of applications to synthetic and real datasets are worked out.

Optimal Investment-Consumption Problem: Post-Retirement with Minimum Guarantee

Hassan Dadashi

Institute for Advanced Studies in Basic Sciences

We study the optimal investment-consumption problem for a member of defined contribution plan during the decumulation phase. For a fixed annuitization time, to achieve higher final annuity, we consider a variable consumption rate. Moreover, to have a minimum guarantee for the final annuity, a safety level for the wealth process is considered. To solve the stochastic optimal control problem via dynamic programing, we obtain a Hamilton-Jacobi-Bellman (HJB) equation on a bounded domain. The existence and uniqueness of classical solutions are proved through the dual transformation. We apply the finite difference method to find numerical approximations of the solution of the HJB equation. Finally, the simulation results for the optimal investment-consumption strategies, optimal wealth process and the final annuity for different admissible ranges of consumption are given. Furthermore, by taking into account the market present value of the cash flows before and after the annuitization, we compare the outcomes of different scenarios.

Dynamic of Futures Price Models in Mathematical Finance

Hamideh Dariush Hamedani

Shahid Beheshti University

In this work, we focus on the futures price models, introduced by Heath and Jara in 1998 in which interest rate contracts are studied as basic securities. We assume that futures prices $F_i(t,T_i)$; $1 \leq i \leq p$ are moving like a linear combination of several independent Brownian motions where the T_i are their maturities. Firstly, we try to get a deep understanding of the correlation structure of the dynamic of these given p futures prices. Secondly, for any given cap $K > 0$, using Principal Component Analysis, we obtain the best m-estimator, $1 \leq m \leq p-1$, for

$$\bar{X} = \left[max\left\{F_1(T_1,T_1), \dots, F_p(T_p,T_p)\right\} - K\right]^+,$$

in $\mathcal{L}^2(\Omega,\mathcal{F},Q)$ - sense where Q is the martingale measure.

Pricing of Callable Put Options with and without Extra Information: A Special Case

Neda Esmaeeli

University of Isfahan

We consider a callable put on a financial market with an underlying following the Black-Scholes model. The callable put is simply a put that can be called by its issuer. We then will place ourselves in the framework of asymmetric information, a situation in which one party has more information compared to other one. We restrict ourselves in this paper to a simple case of information asymmetry and we will employ an algorithm for pricing callable put options with and without this extra information.

Weak Solutions, here, there, everywhere

Arash Fahim

Florida State University

We shall survey the importance of weak solutions in stochastic control in conjunction with historical examples that formed our understanding of weak Solutions. We discuss the remedies that pathwise stochastic control offers to further develop dynamic programming equations when only existence of weak solution is guaranteed.

Multiplicative Ergodic Theorem on the Fields of Banach Spaces

Mazyar Ghani Varzaneh

Technical University of Berlin

Sharif University of Technology

The Multiplicative Ergodic Theorem (MET) is a powerful tool with various applications in different fields of mathematics, including analysis, probability theory, and geometry, and a cornerstone in smooth ergodic theory. It was first proved by Oseledets for matrix cocycles, since then, the theorem attracted many researchers to provide new proofs and new formulations in other settings.

In this talk motivated by our models in stochastic delay equations and stochastic partial differential equations (SPDE), we will present a version of MET for stationary compositions on a (possibly random) field of (potentially distinct) Banach spaces, depending on the random sample. (Joint work with Sebastian Riedel)

Doeblin Trees

Mir-Omid Haji-Mirsadeghi

Sharif University of Technology

This talk is centered on the random graph generated by a Doeblin-type coupling of discrete time processes on a countable state space whereby when two paths meet, they merge. This random graph is studied through a novel subgraph, called a bridge graph, generated by paths started in a fixed state at any time. The bridge graph is made into a unimodular network by marking it and selecting a root in a specified fashion. The unimodularity of this network is leveraged to discern global properties of the larger Doeblin graph. Bi-recurrence, i.e., recurrence both forwards and backwards in time, is introduced and shown to be a key property in uniquely distinguishing paths in the Doeblin graph, and also a decisive property for Markov chains indexed by Z.

Some Recent Results on Stochastic Functional Evolution Equations

Ruhollah Jahanipur

University of Kashan

In this talk, we study a class of semilinear stochastic functional evolution equations with finite delay in which the nonlinearity satisfies the weak condition of demicontinuity with respect to functional variable and also a semimonotone condition. We prove the existence, uniqueness and measurability of the mild solutions based on an extension of the analogous results for non-delay initial value problems on Banach spaces and a version of recently developed random Schauder's fixed point theorem. This will be done first for the generalized solutions and then we follow an approximating procedure to obtain same results for the mild solutions.

A Stochastic Gronwall Lemma and Wellposedness of Path-Dependent SDEs Driven by Martingale Noise

Sima Mehri

University of Warwick

We show existence and uniqueness of solutions of stochastic path-dependent differential equations driven by cadlag martingale noise under joint local monotonicity and coercivity assumptions on the coefficients with a bound in terms of the supremum norm. In this set-up, the usual proof using the ordinary Gronwall lemma together with the Burkholder-Davis-Gundy inequality seems impossible. In order to solve this problem, we state a new and quite general stochastic Gronwall lemma for cadlag martingales using Lenglart's inequality.

Stochastic processes with semi-long range dependence

Farzad Sabzikar

Iowa State University

Stochastic processes with long-range dependence correlation have proven helpful in many areas, from engineering to science in theory and applications. This class includes fractional Brownian motion, fractional Gaussian noise, and fractional ARIMA time series. One of the main properties of long-range dependence is that the spectral density is unbounded at the origin. However, in many applications, data fit with this spectral density model only up to a low-frequency cutoff, after which the observed spectral density remains bounded. This talk presents a novel modification of these models that involves tempering the power-law correlation function with an exponential. This results in a tempered fractional Brownian motion, a tempered fractional Gaussian noise, and a tempered ARIMA time series. These processes have semi-long range dependence: Their autocovariance function resembles that of a long memory model for moderate lags but eventually diminishes exponentially fast according to the presence of a decay factor governed by a tempering parameter. Several theoretical contributions and applications of these new models in finance and geophysics will be presented.

The Deterministic Limit of the Stochastic Process of Recombination

Majid Salamat

Isfahan University of Technology

Genetic information is encoded in terms of finite sequences. Gametes carry one such sequence. Their life cycle is as follows: At fertilisation, two gametes are chosen randomly and unite, thus making a new individual, including both the maternal and the paternal sequence. This process may include recombination, that is, the maternal and paternal sequences perform one or more crossovers and are cut and relinked accordingly, so that two 'mixed' sequences emerge. These are the new gametes and start the next round of fertilisation (by random mating within a large population).

Models of this process aim at describing the dynamics of the genetic composition of a population that goes through this life cycle repeatedly. These models are introduced in different forms: discrete or continuous time; with various assumptions about the crossover pattern; and in a deterministic or a stochastic formulation, depending on whether or not the population is assumed to be so large that stochastic fluctuations may be neglected. In this talk I employ the deterministic continuous-time approach.

Hedging under Uncertainty

Erfan Salavati

Amirkabir University of Technology

One of the fundamental assumptions of the classical mathematical finance is that the market participants evaluate the future events with respect to a single probability measure. This assumption has been challenged by various behavioral experiments. A new alternative is the concept of uncertainty. The mathematical formulation of the uncertainty considers a family of probability measures and then assumed that the worst case happens, i.e., the probability measure with respect to which the risk is maximum.

In the case of static hedging, this results in a robust optimization problem, which is well studied in the literature. But in the case of dynamic hedging, one needs a completely novel idea in order to make sense of the stochastic processes and in particular Brownian motion under uncertainty. This has been achieved by the recently introduced theory of G-Brownian motion.

In this talk we will present a new framework for hedging contingent claims in the Black-Scholes model driven by G-Brownian motion.

Reinforcement Learning in Stochastic Differential Equations

Mohamad Kazem Shirani Faradonbeh

University of Georgia

Study of reinforcement learning policies for unknown diffusion processes is a canonical problem. The goals consist of design and analysis of efficient algorithms for both minimizing the cost function, as well as for accurate estimation of underlying linear stochastic differential equations. A fundamental challenge is to address the exploration-exploitation dilemma. That is, estimation accuracy is necessary for optimal decisions, while sub-optimal decisions are required for obtaining accurate estimates. We present a fast implementable reinforcement learning algorithm and theoretically establish its efficiency. The proposed algorithm learns the true diffusion process and optimal decisions fast, such that the per-unit-time regret decays with the square-root rate as time grows. Further, we present tight results for assuring system stability and for specifying fundamental limits of sub-optimalities caused by uncertainties.

Multi-Class Advance Patient Scheduling

Mohamad Sadegh Shirani Faradonbeh
Stanford University

The problem of advance scheduling of service appointments for patients arriving to a healthcare facility received a lot of attentions in the literature of operations management. Broadly speaking, the main goal is on the efficient assignment of patients entering the system to the next operating days, advance in time and in a dynamic manner. In particular, the problem of multi-class advance patient scheduling, that aims to incorporate differences in the priority levels of patient classes, is of interest in many situations. In this setting, one needs to address important challenges to efficiently utilize the limited and costly resources of the underlying healthcare facilities. Furthermore, a reliable scheduling policy needs to reserve sufficient capacity for high-priority patients, in order to avoid long waiting-times for urgent cases in the future. Accordingly, at every time instant, the policy needs to consider all outstanding appointments, as well as uncertainties in the future demand. This work presents the first theoretically tractable framework for design and analysis of efficient advance scheduling policies in a multi-class setting. First, we provide a realistic formulation of the problem that reflects both the transient as well as the long-term behavior of scheduling policies. Then, we study optimal policies that efficiently schedule patients of different classes and characterize the resulting coarse-grained fluid dynamics, as well as the finer dynamics of diffusion approximation. In fact, the former yields to a simple policy that schedules all patients on the day of their arrival, and also sets the stage for the analysis of the latter stochastic dynamical model. Then, we proceed towards considering diffusion processes based on which the study of scheduling policies becomes a Brownian control problem. Finally, by leveraging a dynamic programming approach, we characterize the optimal policy and validate it through numerical implementations.

Recursive Integral Equations for Random Weights Averages: Exponential Functions and Cauchy Distribution

Ahmad Reza Soltani

Kuwait University

In this article, firstly, we prove that the functions $\phi(x) = e^{cx} I_{(-\infty,0)}(x) + e^{bx} I_{[0,+\infty)}(x)$, c, b constants, are the only solutions to the integral equation $\phi(x) = \int_0^1 \phi(ux)\phi\big((1-u)x\big)du$. This indeed gives the result of Van Asshe (1987) who used the Schwartz distribution theory to prove that for i.i.d X and Y, $UX + (1-U)Y \sim X$ if and only if X has a Cauchy distribution. Secondly, by looking into certain recursive integral equations involving characteristic functions, we prove that if for an $n \geq 2$, the random weight mean $U_{(1)}X_1 + (U_{(2)} - U_{(1)})X_2 + \cdots + (1 - U_{(\mathrm{n}-1)})X_{\mathrm{n}}$ has a Cauchy distribution, then X_1 has a Cauchy distribution; random variables $X_1, \ldots, X_{\mathrm{n}}$ are i.i.d, the random weights are the cuts of $(0, 1)$ by a uniform sample. The multivariate analogue of this result is also provided.

(This research is supported by the Kuwait University, Research Administration, under the Research Project SS-02-19.)

Reaction-Diffusion Equations with Polynomial Drifts Driven by Fractional Brownian Motions

Shiva Zamani

Sharif University of Technology

A reaction-diffusion equation on $[0,1]^d$ with the heat conductivity $k > 0$, a polynomial drift term and an additive noise, fractional in time with $H > 1/2$, and colored in space, is considered. We show the existence, uniqueness and uniform boundedness of solution with respect to k. Also, we show that if k tends to infinity, then the corresponding solutions of the equation converge to a process satisfying a stochastic ordinary differential equation.